AAPT UNITED STATES PHYSICS TEAM
AIP 2009

Semifinal Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts.
- Part A has four questions and is allowed 90 minutes.
- Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 3 parts: the cover sheet (page 2), Part A (pages 3-4), and Part B (pages 6-7). Examinees should be provided parts A and B individually, although they may keep the cover sheet.
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after March 31, 2009.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDAs or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.
- Please provide the examinees with graph paper for Part A.

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Semifinal Exam

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your school ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  School ID #

  Doe, Jamie

  A1 - 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDAs or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after March 31, 2009.**

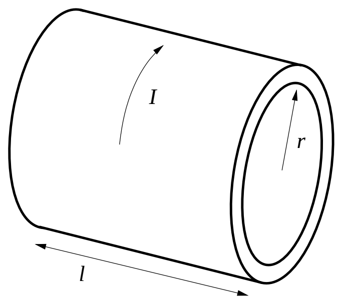
Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9$ N $\cdot$ m^2/C^2	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}$ T $\cdot$ m/A
$c = 3.00 \times 10^8$ m/s	$k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K
$N_\mathrm{A} = 6.02 \times 10^{23}$ (mol)$^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31$ J/(mol $\cdot$ K)
$\sigma = 5.67 \times 10^{-8}$ J/(s $\cdot$ m^2 $\cdot$ K^4)	$e = 1.602 \times 10^{-19}$ C
1eV $= 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}$ J $\cdot$ s $= 4.14 \times 10^{-15}$ eV $\cdot$ s
$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2	$(1+x)^n \approx 1 + nx$ for $\lvert x\rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

Question A1

A hollow cylinder has length l, radius r, and thickness d, where $l \gg r \gg d$, and is made of a material with resistivity ρ. A time-varying current I flows through the cylinder in the tangential direction. Assume the current is always uniformly distributed along the length of the cylinder. The cylinder is fixed so that it cannot move; assume that there are no externally generated magnetic fields during the time considered for the problems below.



a. What is the magnetic field strength B inside the cylinder in terms of I, the dimensions of the cylinder, and fundamental constants?

b. Relate the emf $\mathcal{E}$ developed along the circumference of the cylinder to the rate of change of the current $\frac{dI}{dt}$, the dimensions of the cylinder, and fundamental constants.

c. Relate $\mathcal{E}$ to the current I, the resistivity ρ, and the dimensions of the cylinder.

d. The current at $t = 0$ is I_0. What is the current $I(t)$ for $t > 0$?

Question A2

A mixture of ^{32}P and ^{35}S (two beta emitters widely used in biochemical research) is placed next to a detector and allowed to decay, resulting in the data below. The detector has equal sensitivity to the beta particles emitted by each isotope, and both isotopes decay into stable daughters.

You should analyze the data graphically. Error estimates are not required.

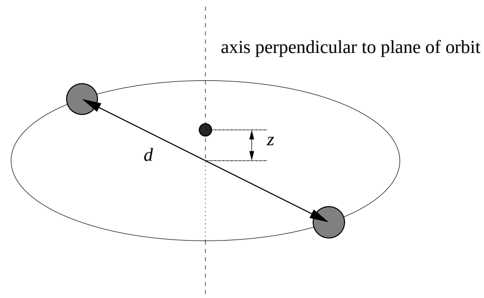
Day	Activity	Day	Activity	Day	Activity
0	64557	40	12441	200	1121
5	51714	60	6385	250	673
10	41444	80	3855	300	467
20	27020	100	2734		
30	18003	150	1626		

a. Determine the half-life of each isotope. ^{35}S has a significantly longer half-life than ^{32}P.

b. Determine the ratio of the number of ^{32}P atoms to the number of ^{35}S atoms in the original sample.

Question A3

Two stars, each of mass M and separated by a distance d, orbit about their center of mass. A planetoid of mass m ($m \ll M$) moves along the axis of this system perpendicular to the orbital plane.

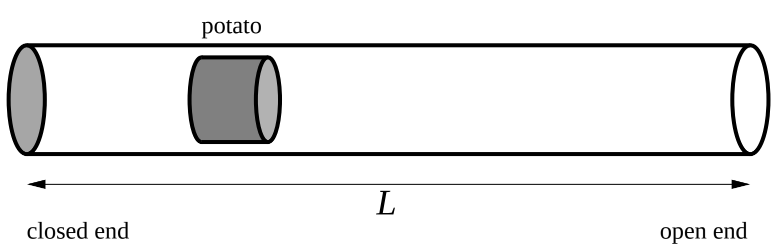


Let T_p be the period of simple harmonic motion for the planetoid for small displacements from the center of mass along the z-axis, and let T_s be the period of motion for the two stars. Determine the ratio T_p/T_s.

This problem was adapted from a problem by French in *Newtonian Mechanics*.

Question A4

A potato gun fires a potato horizontally down a half-open cylinder of cross-sectional area A. When the gun is fired, the potato slug is at rest, the volume between the end of the cylinder and the potato is V_0, and the pressure of the gas in this volume is P_0. The atmospheric pressure is P_{atm}, where $P_0 > P_{\text{atm}}$. The gas in the cylinder is diatomic; this means that $C_{\text{v}} = 5R/2$ and $C_{\text{p}} = 7R/2$. The potato moves down the cylinder quickly enough that no heat is transferred to the gas. Friction between the potato and the barrel is negligible and no gas leaks around the potato.



The parameters P_0, P_{atm}, V_0, and A are fixed, but the overall length L of the barrel may be varied.

a. What is the maximum kinetic energy E_{max} with which the potato can exit the barrel? Express your answer in terms of P_0, P_{atm}, and V_0.

b. What is the length L in this case? Express your answer in terms of P_0, P_{atm}, V_0, and A.

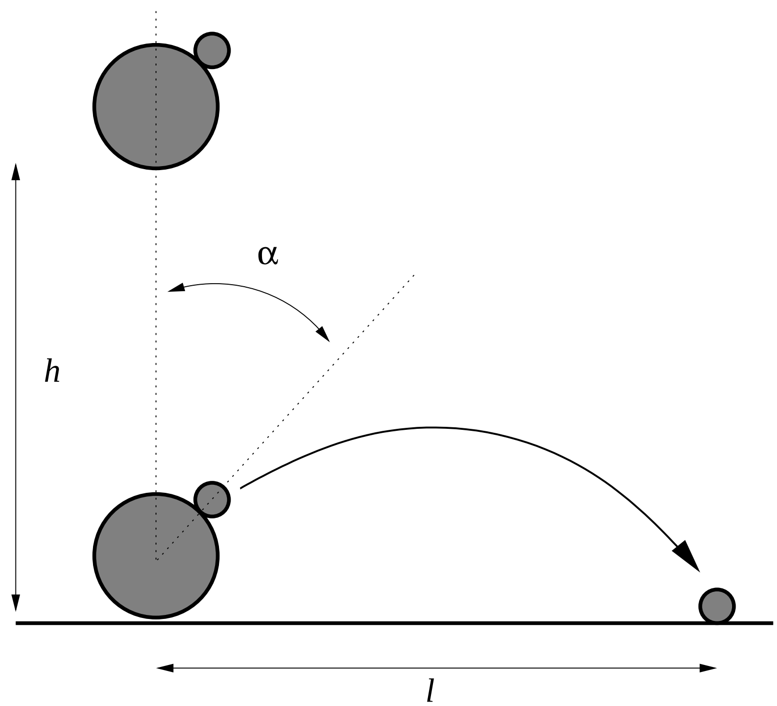
STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

Question B1

A bowling ball and a golf ball are dropped together onto a flat surface from a height h. The bowling ball is much more massive than the golf ball, and both have radii much less than h. The bowling ball collides with the surface and immediately thereafter with the golf ball; the balls are dropped so that all motion is vertical before the second collision, and the golf ball hits the bowling ball at an angle α from its uppermost point, as shown in the diagram. All collisions are perfectly elastic, and there is no surface friction between the bowling ball and the golf ball.



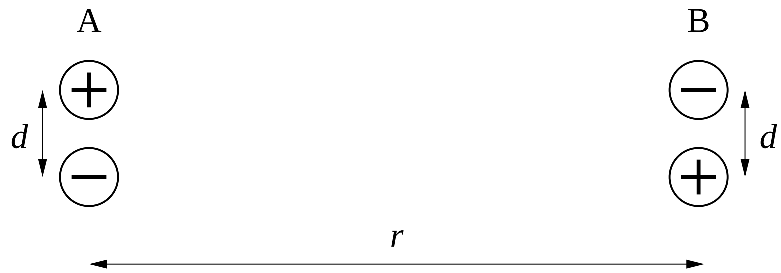
After the collision the golf ball travels in the absence of air resistance and lands a distance l away. The height h is fixed, but α may be varied. What is the maximum possible value of l, and at what angle α is it achieved?

You may present your results as decimals, but remember that you are not allowed to use graphical or algebraic functions of your calculator.

Question B2

An *electric dipole* consists of two charges of equal magnitude q and opposite sign, held rigidly apart by a distance d. The *dipole moment* is defined by $p = qd$.

Now consider two identical, oppositely oriented electric dipoles, separated by a distance r, as shown in the diagram.



a. It is convenient when considering the interaction between the dipoles to choose the zero of potential energy such that the potential energy is zero when the dipoles are very far apart from each other. Using this convention, write an exact expression for the potential energy of this arrangement in terms of q, d, r, and fundamental constants.

b. Assume that $d \ll r$. Give an approximation of your expression for the potential energy to lowest order in d. Rewrite this approximation in terms of only p, r, and fundamental constants.

c. What is the force (magnitude and direction) exerted on one dipole by the other? Continue to make the assumption that $d \ll r$, and again express your result in terms of only p, r, and fundamental constants.

d. What is the electric field near dipole B produced by dipole A? Continue to make the assumption that $d \ll r$ and express your result in terms of only p, r, and fundamental constants.