

3. Transmission Line Modelling 传输线模型

Consider a high-frequency signal propagating along a transmission line. The line is modeled as a series of infinitesimal segments of length Δx (See Fig 3-1). Let L and R be the series inductance and resistance per unit length, and G and C be the shunt conductance and capacitance per unit length, respectively. Assume L, R, G , and C are frequency-independent constants.

考虑高频信号在传输线中的传播。我们将传输线建模为长度为 Δx 的无穷小段的集合 (见图 3-1)。令 L 和 R 分别为单位长度的串联电感和电阻， G 和 C 分别为单位长度的并联电导和电容。假设 L, R, G, C 均为与频率无关的常数。

(ai) [4] By applying Kirchhoff's laws to the infinitesimal circuit model as shown in Fig. 3-1, derive the two coupled partial differential equations relating current $i(x, t)$ and voltage $v(x, t)$ as $\Delta x \rightarrow 0$.

(ai) [4] 对图 3-1 的电路模型应用基尔霍夫定律，推导出当 $\Delta x \rightarrow 0$ 时，描述电流 $i(x, t)$ 与电压 $v(x, t)$ 关系的两个耦合偏微分方程。

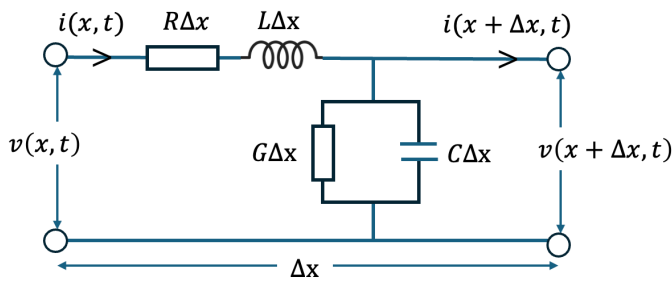


Fig 3-1

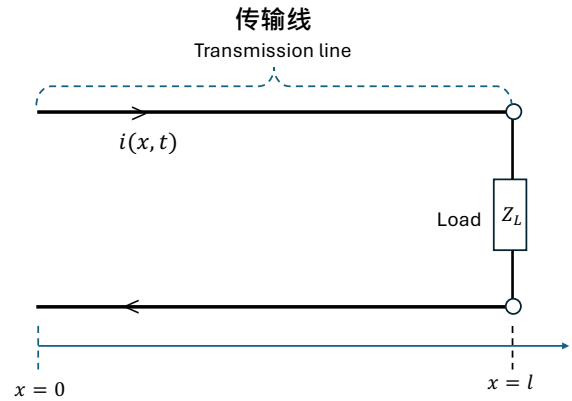


Fig 3-2

(aii) [1] For a sinusoidal signal of angular frequency ω , define $i(x, t) = \text{Re}(I(x)e^{j\omega t})$ and $v(x, t) = \text{Re}(V(x)e^{j\omega t})$, the equations reduce to:

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x), \quad \frac{d^2 V(x)}{dx^2} = \gamma^2 V(x)$$

Find the complex propagation constant γ^2 in terms of L, R, G, C , and ω . Here $j = \sqrt{-1}$ is the imaginary number and $\text{Re}(z)$ is the real part of a complex number z .

(aii) [1] 对于角频率为 ω 的正弦信号，设 $i(x, t) = \text{Re}(I(x)e^{j\omega t})$ 且 $v(x, t) = \text{Re}(V(x)e^{j\omega t})$ ，方程可化简为：

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x), \quad \frac{d^2 V(x)}{dx^2} = \gamma^2 V(x)$$

求复传播常数 γ^2 关于 L, R, G, C 和 ω 的表达式。这里 $j = \sqrt{-1}$ 是虚数， $\text{Re}(z)$ 表示复数 z 的实部。

(aiii) [1] The general solutions are $I(x) = I_0^+ e^{-\gamma x} + I_0^- e^{\gamma x}$ and $V(x) = V_0^+ e^{-\gamma x} + V_0^- e^{\gamma x}$. Define the characteristic impedance $Z_0 = \left| \frac{V_0^+}{I_0^+} \right| = \left| \frac{V_0^-}{I_0^-} \right|$. Determine Z_0 in terms of L, R, G, C , and ω .

(aiii) [1] 已知通解为 $I(x) = I_0^+ e^{-\gamma x} + I_0^- e^{\gamma x}$ 和 $V(x) = V_0^+ e^{-\gamma x} + V_0^- e^{\gamma x}$ 。定义特性阻抗为 $Z_0 = \left| \frac{V_0^+}{I_0^+} \right| = \left| \frac{V_0^-}{I_0^-} \right|$ 。求 Z_0 的表达式，用 L, R, G, C 和 ω 表达。

(aiv) [2] Find the condition relating L, R, G and C that allows a signal to propagate without distortion. Determine the phase velocity v_p in this case.

(aiv) [2] 试求当 L, R, G, C 满足何种关系时，信号可实现无失真传播？并求出该情形下的相速度 v_p 。

(bi) [1] In Fig. 3-2, a transmission line is terminated by a load impedance Z_L . When an incident signal V_i travels along the line, a portion is reflected as V_r . Derive the voltage reflection coefficient, $\Gamma = \frac{V_r}{V_i}$, in terms of Z_L and the characteristic impedance Z_0 .

(bi) [1] 如图 3-2 所示，传输线终端接有负载阻抗 Z_L 。当入射信号 V_i 沿传输线传播时，部分信号将被反射，形成反射波 V_r 。试推导电压反射系数 $\Gamma = \frac{V_r}{V_i}$ 的表达式（用 Z_L 和特性阻抗 Z_0 表示）。

(bii) [1] State the required condition for the incident voltage signal V_i to undergo a π phase change upon reflection.

(bii) [1] 若要使入射电压信号 V_i 在反射时产生 π 的相位变化，负载阻抗 Z_L 需满足什么条件？