2. [10] Rods and Planes 棒与面

(a) Gravitational Field Generated by Rods 由细棒产生的引力场

(ai) [1] Find the magnitude of gravitational field at a distance d from an infinitely long thin rod with mass per unit length λ.

(aii) [3] Consider 2026 infinitely long thin rods oriented along the z axis, each with mass per unit length λ. The 2026 rods are on positions $(x_n, y_n) = \left(r_0 \cos\frac{2n\pi}{2026}, r_0 \sin\frac{2n\pi}{2026}\right)$, where $0 \le n \le 2025$. Find the net force on a point mass with mass m located at $(d, 0)$. (Hint: You can consider the complex roots of the polynomial: $(z+d)^{2026} - r_0^{2026} = 0$)

(aiii) [1] Suppose the small point mass is constrained to only move along the x axis, determine if it will oscillate around the origin. (Just answer yes or no)

(ai) [1] 求距离一根线质量密度为 λ 的"無限长细棒"距离 d 处的引力场强度。

(aii) [3] 考虑 2026 根沿 z 方向排列的無限长细棒，每根细棒的质量线密度皆为 λ。第 n 根细棒在平面上的位置为 $(x_n, y_n) = \left(r_0 \cos\frac{2n\pi}{2026}, r_0 \sin\frac{2n\pi}{2026}\right)$。求质量为 m 的质点在 $(d, 0)$ 点的合外力。

（提示：你可以考虑多项式$(z+d)^{2026} - r_0^{2026} = 0$ 的复根）

(aiii) [1] 若该质点仅被限制在 x 轴运动，判斷其会否在原点附近來回移动。（只需回答"是"或"否"）

(b) Gravitational Field Generated by a Plate. 由平板产生的引力场

(bi) [1] Consider an infinite horizontal plane with mass per unit area σ, find the magnitude of gravitational field a distance h above the plate.

(bii) [2] Consider a flat and horizontal circular plate with radius R and total mass m. Find the magnitude of gravitational field due to the plate at a distance $h = R$ above the center of the plate if the plate has uniform density.

(Hint: $\int_0^1 \frac{x\mathrm{d}x}{(1+x^2)^{\frac{3}{2}}} = 1 - \frac{\sqrt{2}}{2}$)

(biii) [2] Consider a horizontal, equilateral triangular plate of mass m, and side length a. Find the magnitude of gravitational field due to the plate at a distance $h = \frac{\sqrt{6}a}{12}$ above the centroid of the plate if the plate has uniform density. (Hint: The distance from the center of a regular tetrahedron to any of its face is $\frac{\sqrt{6}}{12}$ of the side length of each face.)

(bi) [1] 考虑一张无限大的水平平板，其面质量密度为 σ，求在距离平板高度为 h 处的引力场强度。

(bii) [2] 考虑一张水平放置的圆形平板，半径为 R，质量平均分佈，总质量为 m。求在距离平板中心高度$h = R$处，由该平板产生的引力场强度。（提示：$\int_0^1 \frac{x\mathrm{d}x}{(1+x^2)^{\frac{3}{2}}} = 1 - \frac{\sqrt{2}}{2}$）

(biii) [2] 考虑一个质量为 m、边长为 a的水平正三角形板。求位于平板质心正上方、高度 $h = \frac{\sqrt{6}a}{12}$ 处的引力场强度。（提示：正四面体的中心到任一面的距离为其边长的$\frac{\sqrt{6}}{12}$。）