7. Water hose (5 points) — *Ralf Robert Paabo.* A liquid with density ρ flows in a pipe with diameter D. At low mean speeds v, the flow is laminar (uniform and without vortices); the drag force per unit area $F_\mathrm{L} = \mu\,\mathrm{d}u/\mathrm{d}r$ on the liquid comes from wall friction, where μ (unit $\mathrm{Pa}\cdot\mathrm{s}$) is the dynamic viscosity, u is the local speed, and r is the distance from the axis. When v is large enough, the flow becomes turbulent, filled with vortices whose velocity fluctuations are of the order of v itself. The drag F_T still comes from wall friction $\mu\,\mathrm{d}u/\mathrm{d}r$, but the vortices squeeze the near-wall layer (across which the flow speed drops from v to zero) to a thickness $\delta \ll D$ where the drag force per unit area $\mu v/\delta$ is of the order of the dynamic pressure ρv^2 carried by the vortices.

i) *(1.5 points)* At any given flow speed, only one of the two mechanisms actually dominates the drag – but the dimensionless ratio of the two characteristic drag scales, $R = F_\mathrm{T}/F_\mathrm{L}$, can be computed regardless and tells us which regime we are in. This ratio is the *Reynolds number*. Express R as a product of powers of ρ, v, D, and μ.

A water pump with output power $P = 250\,\mathrm{W}$ is used for watering a garden; it draws water from a depth $h = 20\,\mathrm{m}$ directly into a hose of length $s = 40\,\mathrm{m}$ and inner diameter $d = 13\,\mathrm{mm}$. Water leaves the end of the hose at a volumetric rate $Q = 25\,\mathrm{L}\cdot\mathrm{min}^{-1}$. Density of water is $\rho_\mathrm{w} = 1000\,\mathrm{kg}\cdot\mathrm{m}^{-3}$, viscosity $\mu_\mathrm{w} = 1.1\times10^{-3}\,\mathrm{Pa}\cdot\mathrm{s}$ and gravitational acceleration is $g = 9.8\,\mathrm{N}\cdot\mathrm{kg}^{-1}$.

ii) *(0.5 points)* Determine the flow type in the hose. It is known that flow is turbulent when $R > 2500$ and laminar otherwise.

iii) *(3 points)* The gardener is cleaning tools by directing the water jet onto dirty surfaces. Where the jet meets the surface, the pressure rises above atmospheric; this excess pressure is what removes dirt. To make cleaning more efficient, the gardener attaches a spray nozzle set to narrow-jet mode, so that the exit cross-section of the nozzle is $f = 15\,\%$ of the hose's cross-section. By what factor does the volumetric flow rate Q change? By what factor does the excess pressure at the dirty surface change? Find both within $1\,\%$ accuracy. You are allowed to use numerical methods.