

3. KIRILL ON A SWING (8 points) —

Kaarel Hänni. A swing has rigid rods attaching it to a horizontal pivot, so it can rotate freely in a vertical plane. Initially the swing hangs vertically below the pivot with Kirill standing upright on the seat; his friend gives him an initial angular velocity ω_0 about the pivot. Kirill is practising athletic swinging: whenever the swing momentarily has zero angular velocity, he quickly squats; whenever the rods are again vertical, he quickly stands up. Treat Kirill as a point mass at distance a from the pivot when standing and b when squatting ($a < b$); squatting and standing are motions along the rods. Neglect friction, air resistance, and the mass of the swing. Gravitational acceleration is g .

i) (1.5 points) Sketch, qualitatively, how Kirill's angular speed depends on time during the first full period of the swing.

ii) (2 points) On the phase diagram (angular velocity vs. angle), sketch, qualitatively, the trajectory from the initial push until the swing first goes over the top. The total number of periods shown need not be correct.

iii) (4.5 points) Find how many times Kirill must stand up before the swing first goes over the top. Evaluate your answer for $a = 2.5\text{ m}$, $b = 3.0\text{ m}$, $\omega_0 = 1.0\text{ rad/s}$, and $g = 10\text{ m/s}^2$.