Problem 3. Let $a_1, a_2, a_3, \ldots$ be an infinite sequence of positive integers, and let N be a positive integer. Suppose that, for each $n > N$, a_n is equal to the number of times a_{n-1} appears in the list $a_1, a_2, \ldots, a_{n-1}$.

Prove that at least one of the sequences $a_1, a_3, a_5, \ldots$ and $a_2, a_4, a_6, \ldots$ is eventually periodic.

(An infinite sequence $b_1, b_2, b_3, \ldots$ is *eventually periodic* if there exist positive integers p and M such that $b_{m+p} = b_m$ for all $m \geqslant M$.)