

Problem 1. Determine all real numbers α such that, for every positive integer n , the integer

$$\lfloor \alpha \rfloor + \lfloor 2\alpha \rfloor + \cdots + \lfloor n\alpha \rfloor$$

is a multiple of n . (Note that $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z . For example, $\lfloor -\pi \rfloor = -4$ and $\lfloor 2 \rfloor = \lfloor 2.9 \rfloor = 2$.)