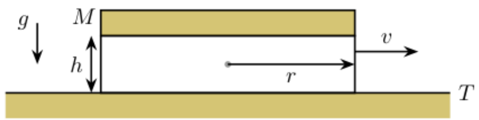
T3: Dry ice hockey (10 pts)

At pressure $p_0 = 100\,\mathrm{kPa}$, solid CO_2 (dry ice) sublimates (goes from solid to gaseous state) at $T_s = -78.5\,°\mathrm{C}$. Its saturated vapour pressure follows the Clausius–Clapeyron relation:

$$\frac{\mathrm{d}p_{\mathrm{sat}}}{\mathrm{d}T} = \frac{\mu\lambda p_{\mathrm{sat}}}{RT^2},$$

where the latent heat of sublimation is $\lambda = 600\,\mathrm{kJ/kg}$, the molar mass is $\mu = 0.044\,\mathrm{kg/mol}$, and the gas constant is $R = 8.3\,\mathrm{J/(mol \cdot K)}$. The thermal conductivity of the CO_2 gas is $\kappa = 10\,\mathrm{mW/(m \cdot K)}$ and its dynamic viscosity is $\eta = 10\,\mathrm{\mu Pa \cdot s}$. The density of dry ice is $\rho = 1500\,\mathrm{kg/m^3}$ and the gravitational acceleration is $g = 10\,\mathrm{m/s^2}$.



A puck of radius $r = 10\,\mathrm{mm}$ consists of a disc of dry ice of thickness $h = 1\,\mathrm{mm}$, and a metal disc of mass $M = 0.01\,\mathrm{kg}$ on top of it. The initial temperature of the puck is T_s and the ambient pressure is p_0.

The puck is placed on a horizontal metal plate which is held at a constant temperature $T = T_s + \Delta T$, and given an initial horizontal velocity $v = 10\,\mathrm{mm/s}$. After a very long time, the horizontal displacement L of the metal disc is measured.

a) **(2 pts)** When ΔT is sufficiently small, L is negligible and independent of ΔT. However, when ΔT reaches a critical value ΔT_c, the function $L(\Delta T)$ starts to grow. Estimate ΔT_c.

b) **(8 pts)** Estimate the maximal value L_{max} of the function $L(\Delta T)$.

Treat the CO_2 gas as ideal. Assume no tilting of the puck at any moment, all surfaces are perfectly smooth, and the thermal conductivities of the metal, dry ice, and gas satisfy $\kappa_{\mathrm{metal}} \gg \kappa_{\mathrm{ice}} \gg \kappa$.