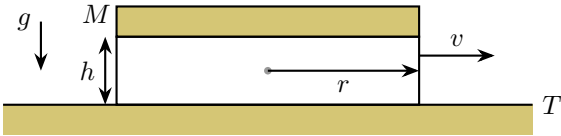


T3: Dry ice hockey (10 pts)

At pressure $p_0 = 100 \text{ kPa}$, solid CO_2 (dry ice) sublimates (goes from solid to gaseous state) at $T_s = -78.5^\circ\text{C}$. Its saturated vapour pressure follows the Clausius-Clapeyron relation:

$$\frac{dp_{\text{sat}}}{dT} = \frac{\mu \lambda p_{\text{sat}}}{RT^2},$$

where the latent heat of sublimation is $\lambda = 600 \text{ kJ/kg}$, the molar mass is $\mu = 0.044 \text{ kg/mol}$, and the gas constant is $R = 8.3 \text{ J/(mol} \cdot \text{K)}$. The thermal conductivity of the CO_2 gas is $\kappa = 10 \text{ mW/(m} \cdot \text{K)}$ and its dynamic viscosity is $\eta = 10 \text{ } \mu\text{Pa} \cdot \text{s}$. The density of dry ice is $\rho = 1500 \text{ kg/m}^3$ and the gravitational acceleration is $g = 10 \text{ m/s}^2$.



A puck of radius $r = 10 \text{ mm}$ consists of a disc of dry ice of thickness $h = 1 \text{ mm}$, and a metal disc of mass $M = 0.01 \text{ kg}$ on top of it. The initial temperature of the puck is T_s and the ambient pressure is p_0 .

The puck is placed on a horizontal metal plate which is held at a constant temperature $T = T_s + \Delta T$, and given an initial horizontal velocity $v = 10 \text{ mm/s}$. After a very long time, the horizontal displacement L of the metal disc is measured.

- (2 pts)** When ΔT is sufficiently small, L is negligible and independent of ΔT . However, when ΔT reaches a critical value ΔT_c , the function $L(\Delta T)$ starts to grow. Estimate ΔT_c .
- (8 pts)** Estimate the maximal value L_{max} of the function $L(\Delta T)$.

Treat the CO_2 gas as ideal. Assume no tilting of the puck at any moment, all surfaces are perfectly smooth, and the thermal conductivities of the metal, dry ice, and gas satisfy $\kappa_{\text{metal}} \gg \kappa_{\text{ice}} \gg \kappa$.